Re-projection without Reconstruction

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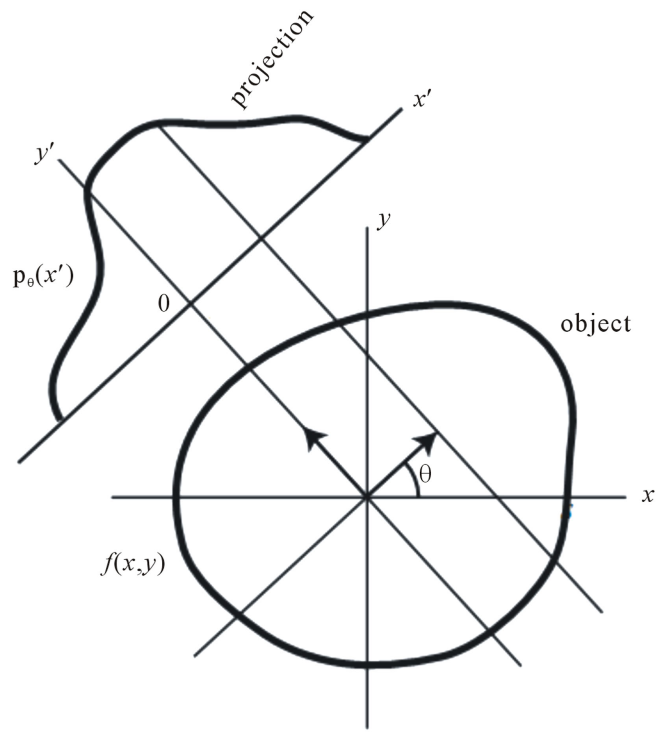
presented by Dimitri Pertin for Mojette Day 2015, February 2015

Introduction

Tomography + Inverse Problem

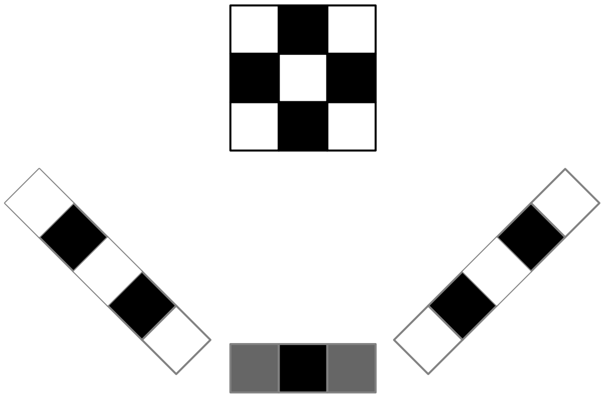
Reconstruct from a sufficient amount of projections



On the determination of functions from their integral values along certain manifolds [Rad86]

Tomography + Inverse Problem

Reconstruct from a sufficient amount of projections

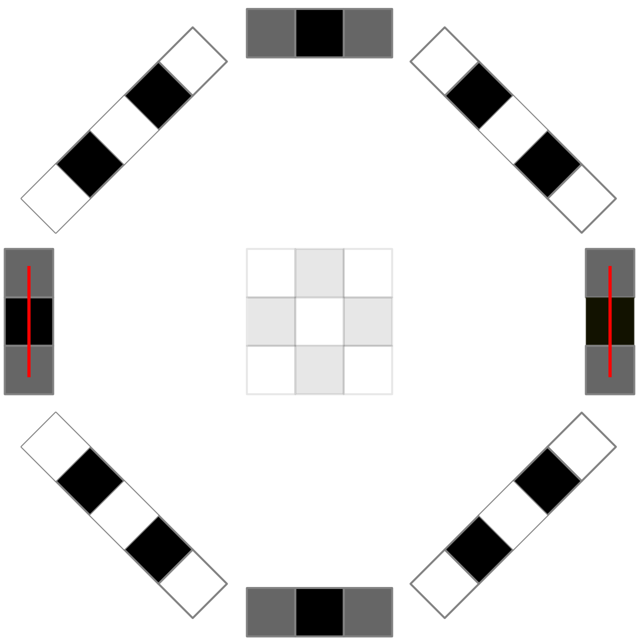


Picture from Wikipedia. *Image representation via a finite Radon transform* [MF93]

Redundant Representations

Redundancy is key to provide fault-tolerance

- Redundancy to face failures
  - e.g. storage systems, transmission
- Once projections are computed, image is not directly accessible
- Restore redundancy when failures occured



Controlled redundancy for image coding and high-speed transmission [NGPB96]

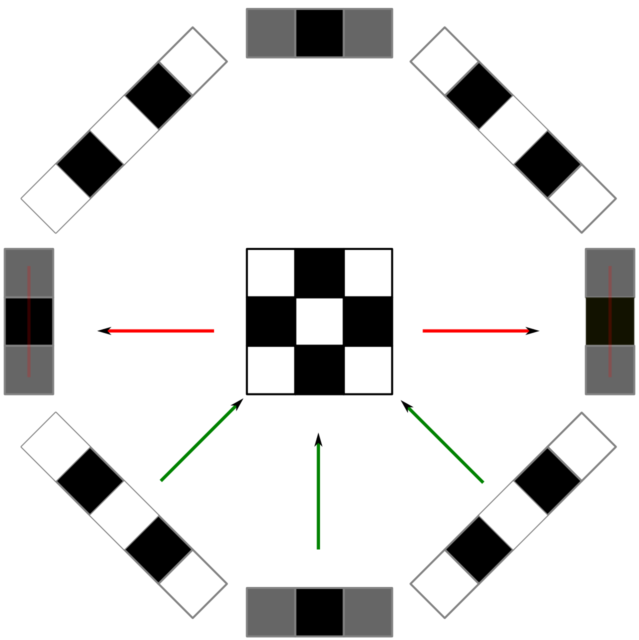
Compute Redundant Projections?

Naïve and Simple way

- Fetch enough projections
- Reconstruct the image
- Re-project along a direction

Why is it bad?

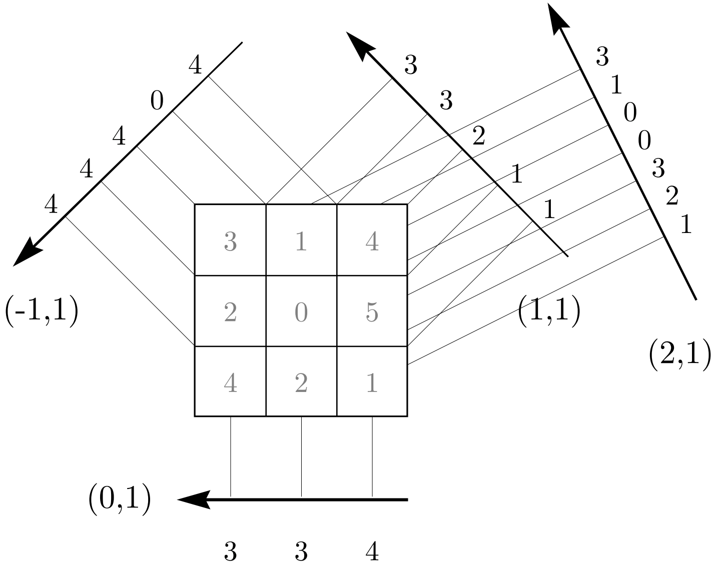
- Reconstruct explicitly the image
  - only the owner should access the image in clear
- Centralize the workload



Mojette Transform

Mojette Transform

Forward Mojette Transform



- Discrete $P \times Q$ image
- Projections along discrete directions (p, q)
- In the example:
  - $(P = 3) \times (Q = 3)$ image
  - $S = \{(2, 1), (1, 1), (0, 1), (-1, 1)\}$
  - addition is done modulo 6
- Projection size: $B(P, Q, p, q) = \ |p|\ (Q-1) + \ |q|\ (P-1) + 1$

$$(M_{(p,q)} f)(b) = \sum_{k=0}^{P-1} \sum_{l=0}^{Q-1} f(k, l)[b = -kq + lp]$$

The Mojette transform. Theory and applications [Gué09]

Mojette Transform

Uniqueness of the Reconstruction

- Condition defined for **rectangular** shapes by Katz criterion:

$$P \leq \sum_i |p_i| \text{ or } Q \leq \sum_i |q_i|$$

- Condition defined for **arbitrary** shapes by *Ghost* properties
  - defined in the next slide...

Questions of uniqueness and resolution in reconstruction from projections [Kat78]

Mojette Transform

Ghosts: invisible image elements

$$G_{\{(p,q)\}} : p \mapsto \begin{cases} 1 & \text{if } p = (0,0) \\ -1 & \text{if } p = (p,q) \\ 0 & \text{otherwise} \end{cases}$$

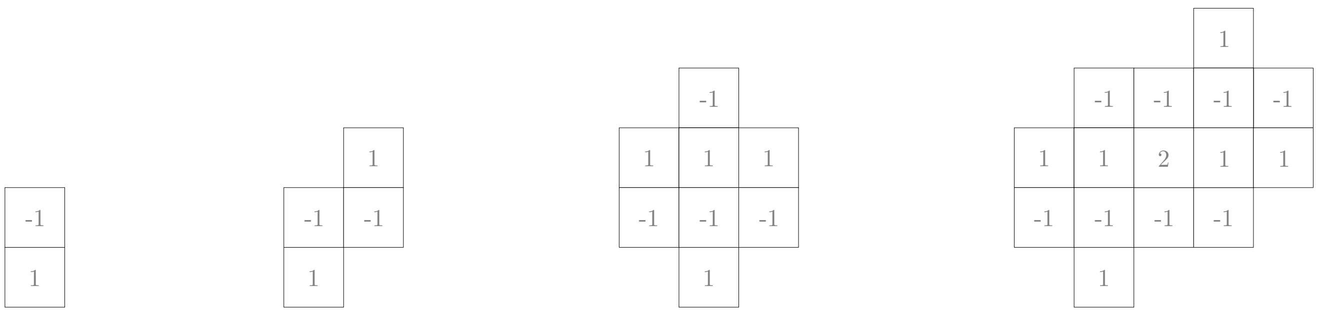
$G_{\{(0,1)\}}$ $G_{\{(1,1)\}}$ $G_{\{(-1,1)\}}$ $G_{\{(2,1)\}}$

La transformée Mojette: une représentation redondante pour l'image [NG98]

Mojette Transform

Composed Ghosts

$$
\begin{aligned}
G_{\{(0,1),(1,1)\}} &= G_{\{(0,1)\}} * G_{\{(1,1)\}} \\
G_{\{(-1,1),(0,1),(1,1)\}} &= G_{\{(-1,1)\}} * G_{\{(0,1),(1,1)\}} \\
G_{\{(2,1),(-1,1),(0,1),(1,1)\}} &= G_{\{(2,1)\}} * G_{\{(-1,1),(0,1),(1,1)\}}
\end{aligned}
$$



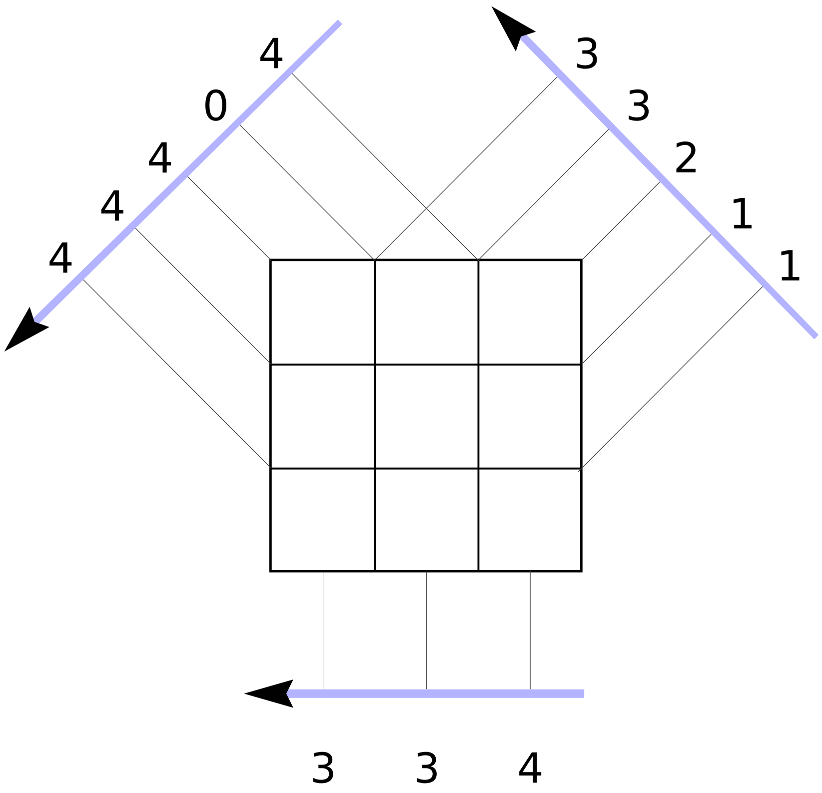
$G_{\{(0,1)\}}$ $\quad$ $G_{\{(0,1),(1,1)\}}$ $\quad$ $G_{\{(-1,1),(0,1),(1,1)\}}$ $\quad$ $G_{\{(-1,1),(0,1),(1,1),(2,1)\}}$

- Any image that has null projections is composed of ghosts

La transformée Mojette: une représentation redondante pour l'image [NG98]

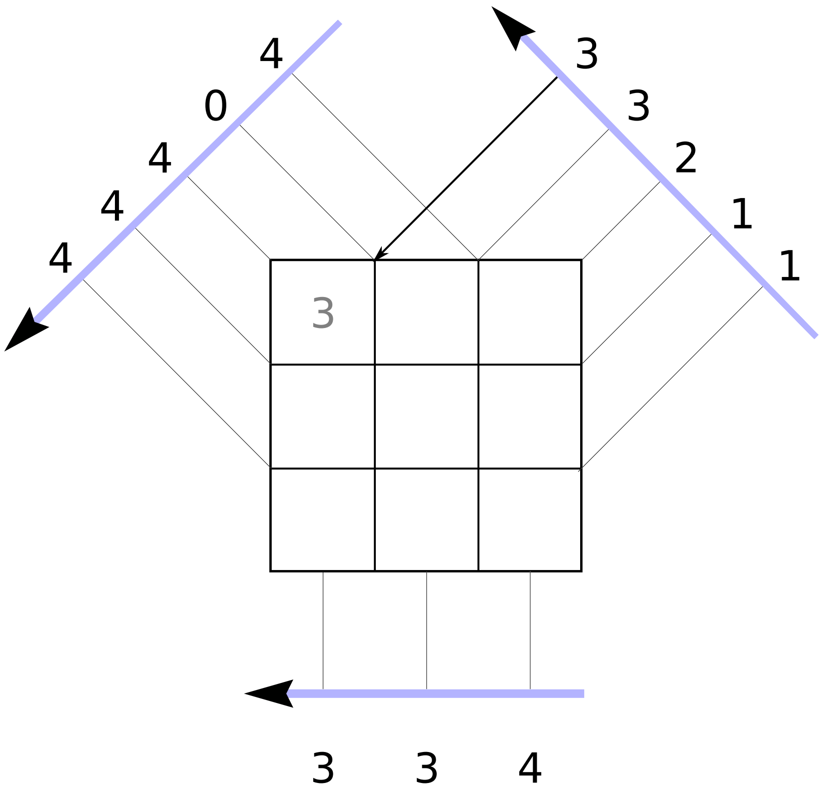
Mojette Transform

Inverse Algorithm



Mojette Transform

Inverse Algorithm



Mojette Transform

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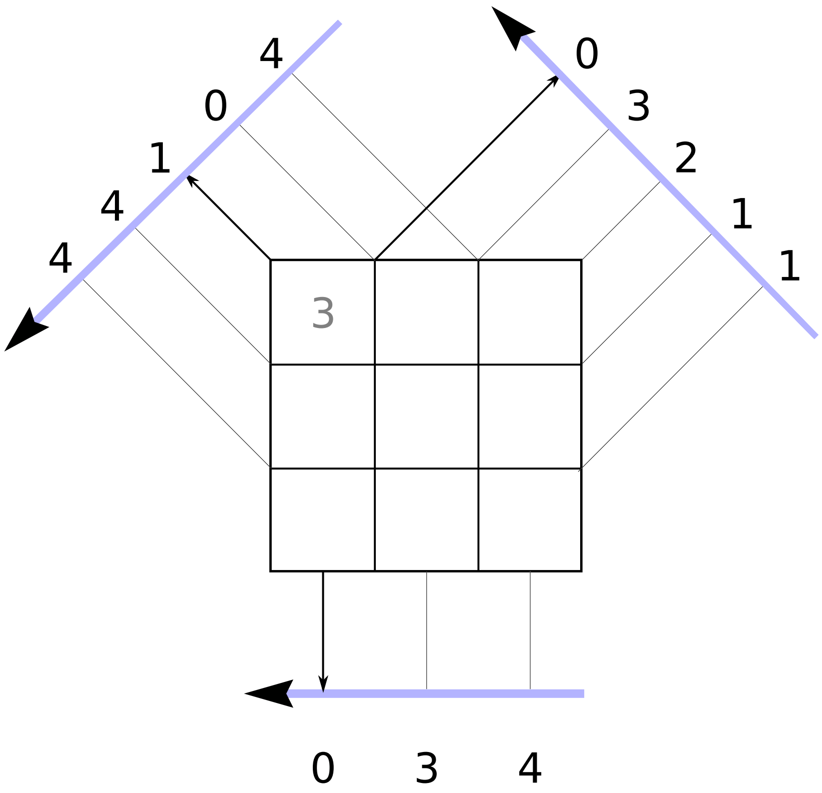

Mojette Transform

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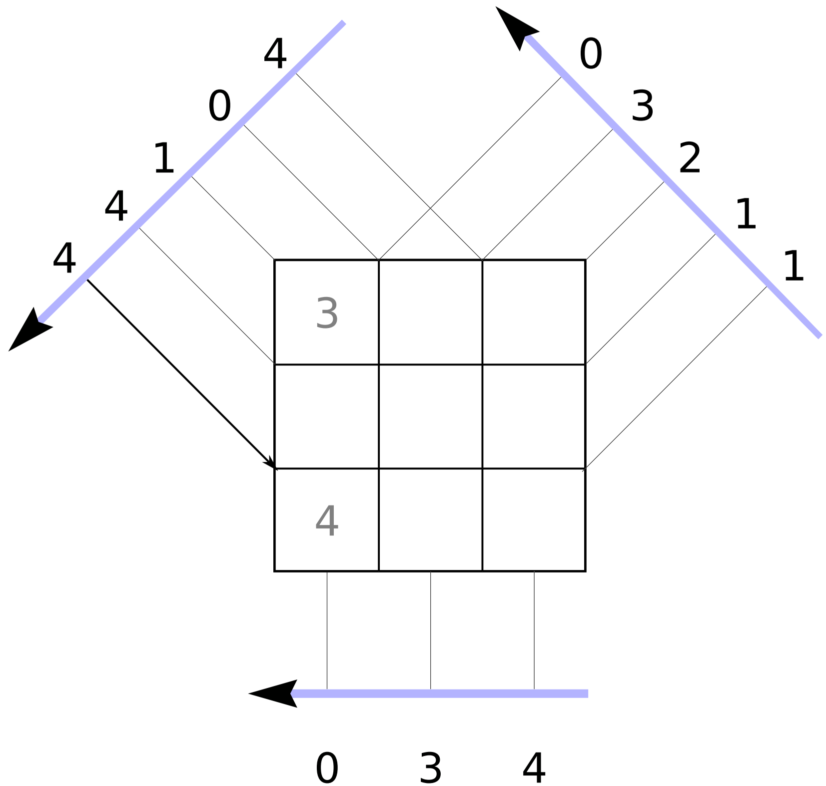

Mojette Transform

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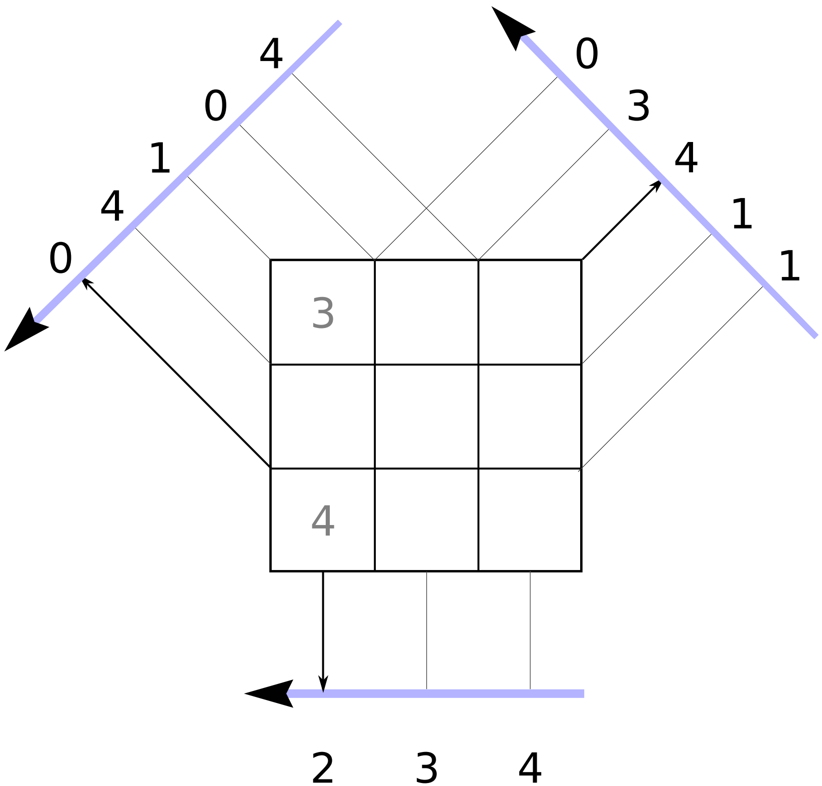

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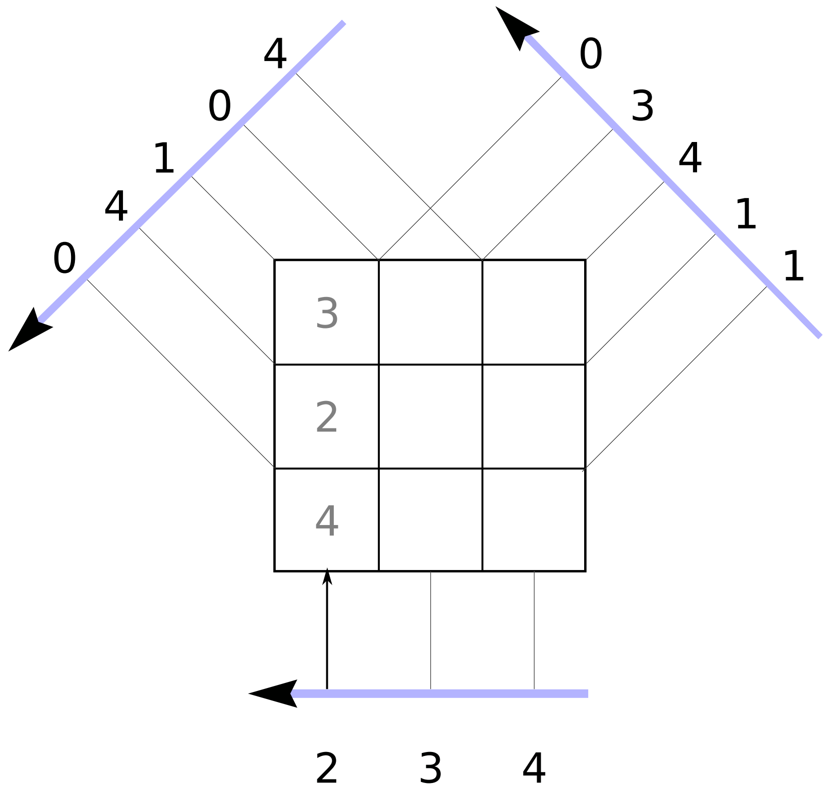

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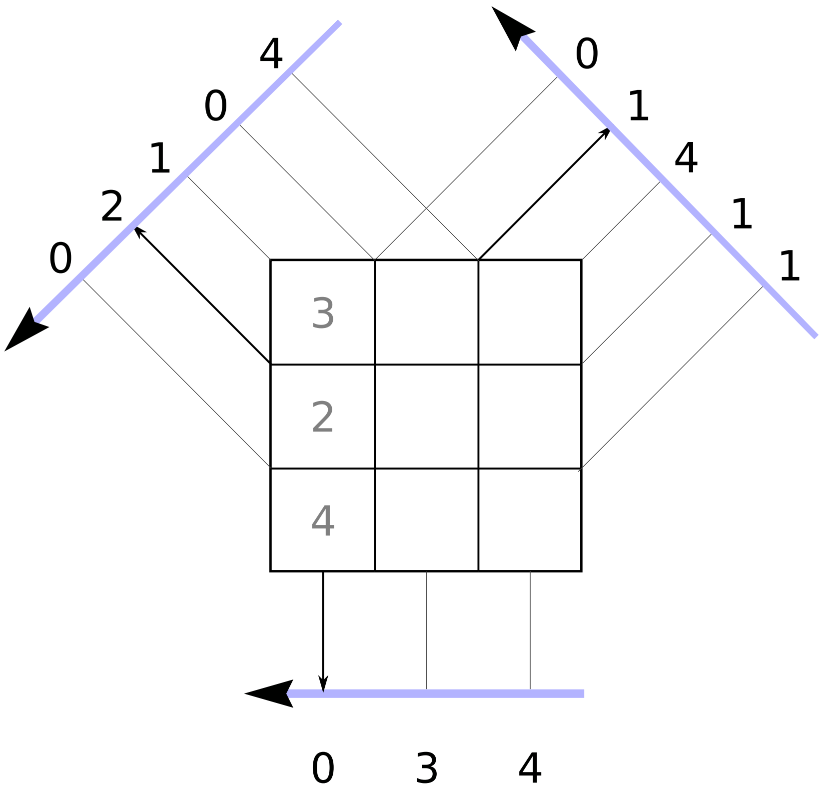

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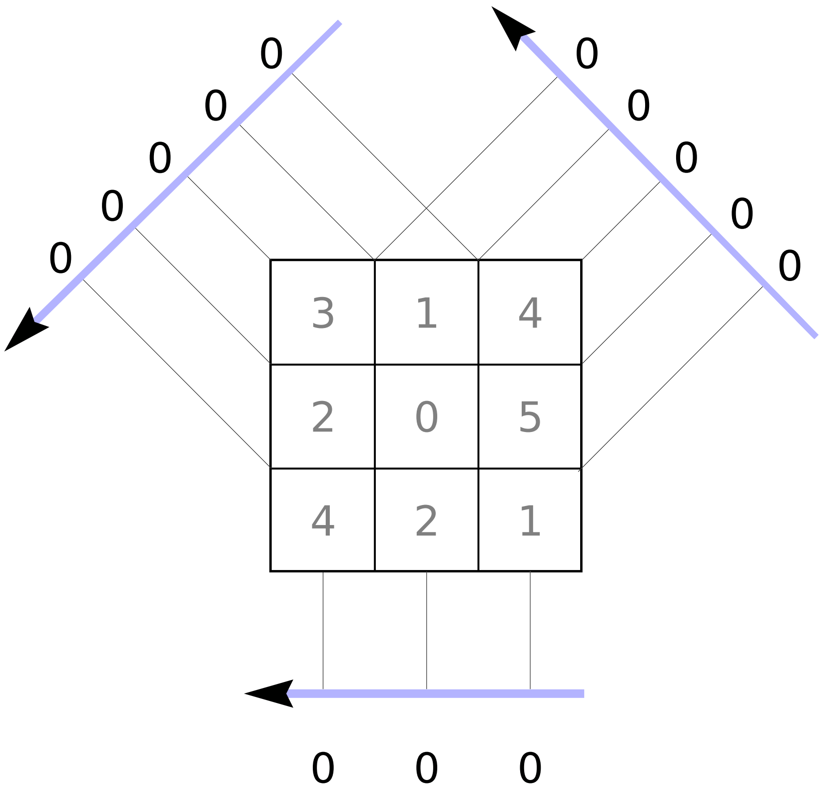


Mojette Re-projection

Mojette Re-projection

Re-projections of Partial Reconstructions

1) Use the linear property to decompose the reconstruction process

- Each projection can individually compute a partial reconstruction $f_S^{\{(p_i, q_i)\}}$ considering the others as null
    - If the subsets R_i form a partition of S then, by linearity, $f = \sum_i f_S^{R_i}$

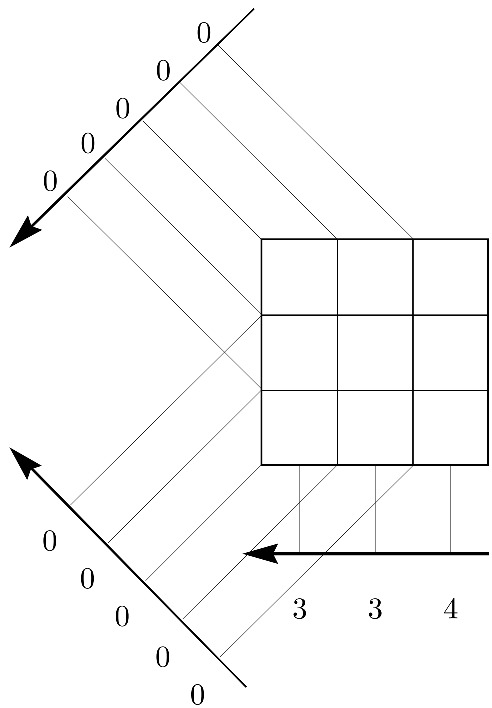
2) Project along a direction (p_k, q_k)

- Project each partial reconstruction along a direction: $M_{(p_k, q_k)} f_S^{\{(p_i, q_i)\}}$
- Sum the re-projections from each partial reconstruction

$$M_{(p_k, q_k)} f = \sum_{R \in P} M_{(p_k, q_k)} f_S^R$$

Mojette Re-projection

Partial Reconstruction $f_S^{\{(0,1)\}}$



Mojette Re-projection

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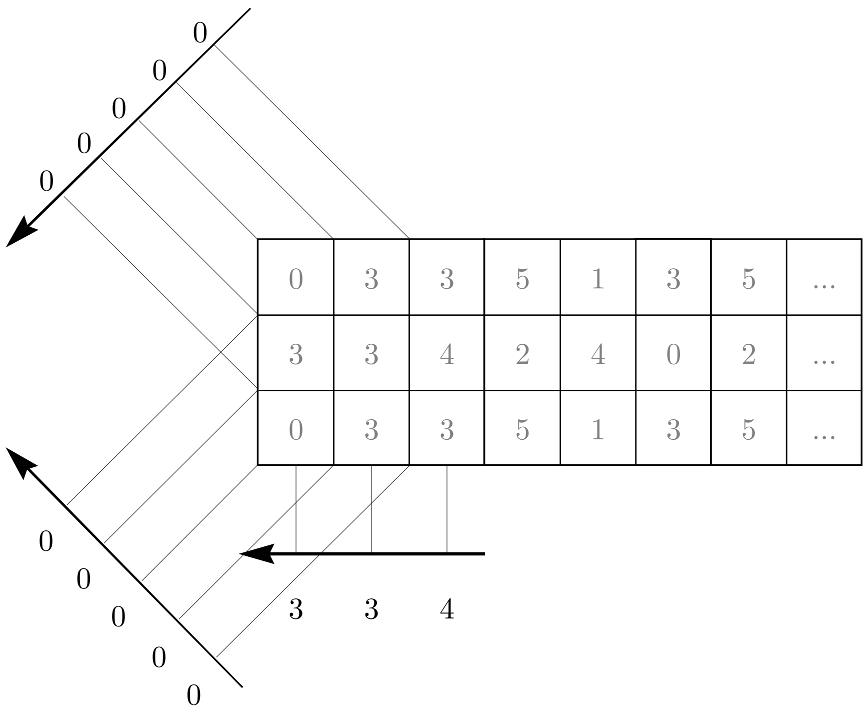

Mojette Re-projection

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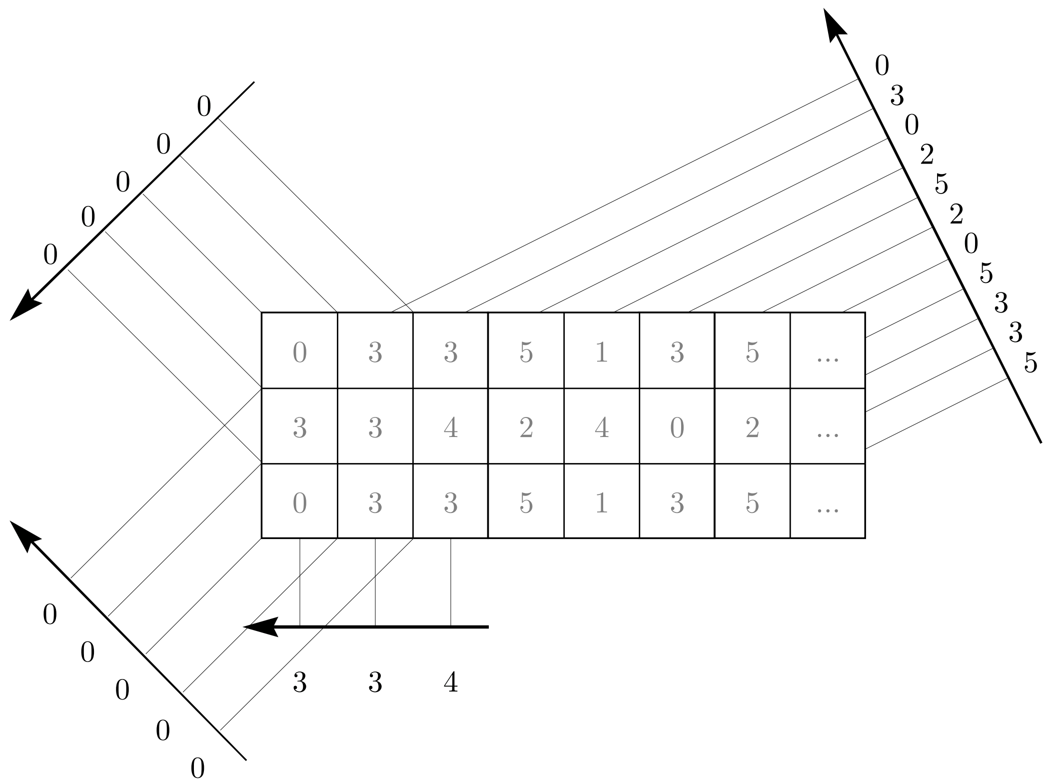

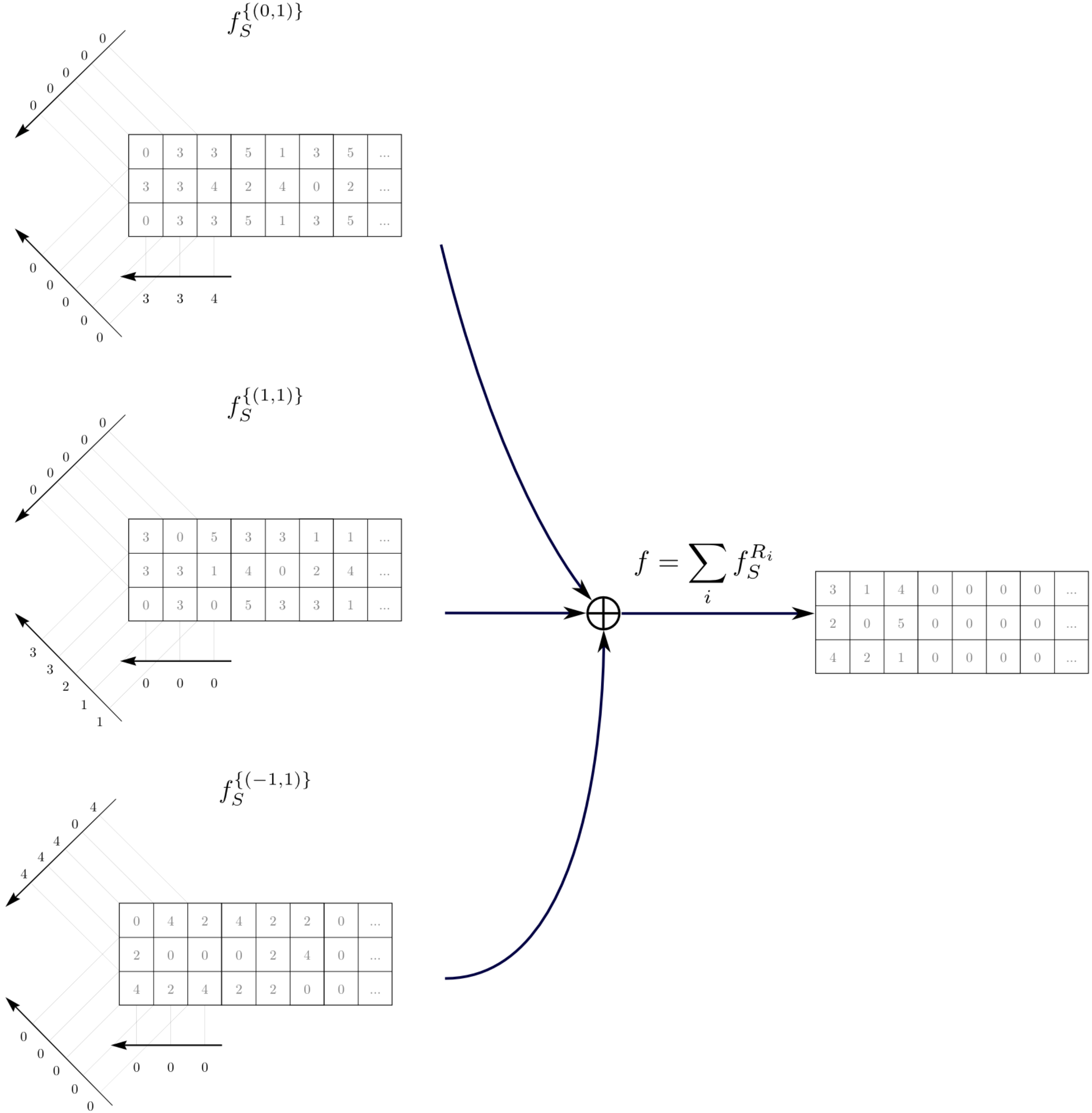

$f_S^{\{(0,1)\}}$
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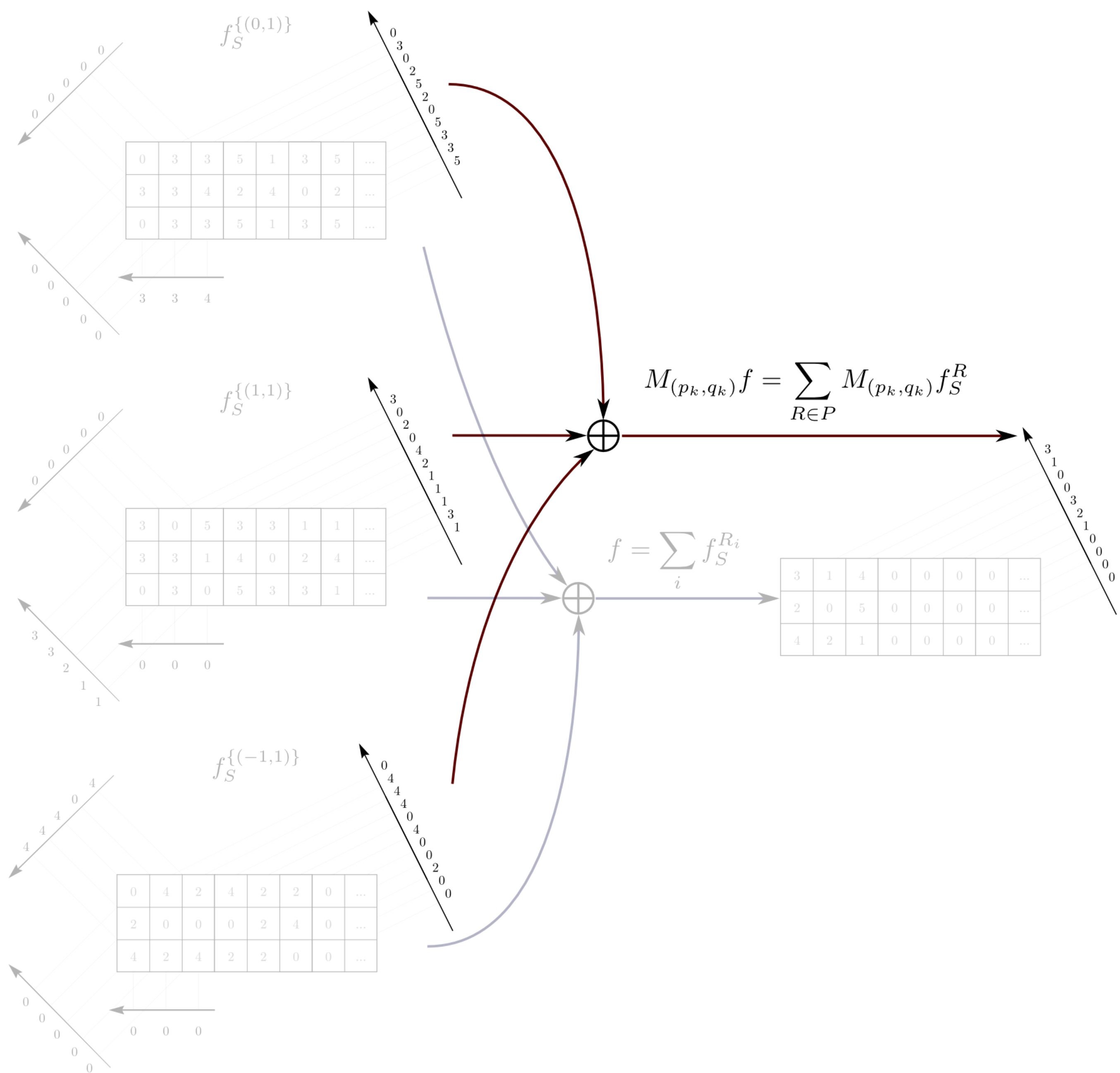
$f_S^{\{(-1,1)\}}$
$f = \sum_i f_S^{R_i}$


$f_S^{\{(0,1)\}}$
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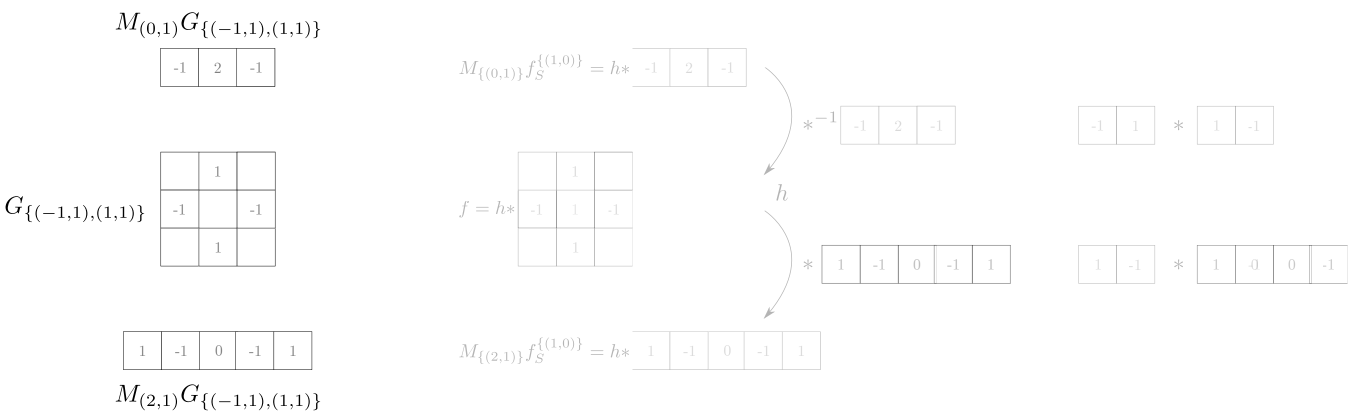
$M_{(p_k,q_k)}f = \sum_{R\in P} M_{(p_k,q_k)} f_S^R$
$f = \sum_i f_S^{R_i}$

Mojette Re-projection

Re-projection by 1D Convolutions

- The partial reconstruction process considers other projections as null, so:
    - $f_S^{\{(p_i,q_i)\}}$ is a ghost for directions $S\backslash\{(p_i,q_i)\}$
    - the partial reconstruction is composed of $G_{S\backslash\{(p_i,q_i)\}}$



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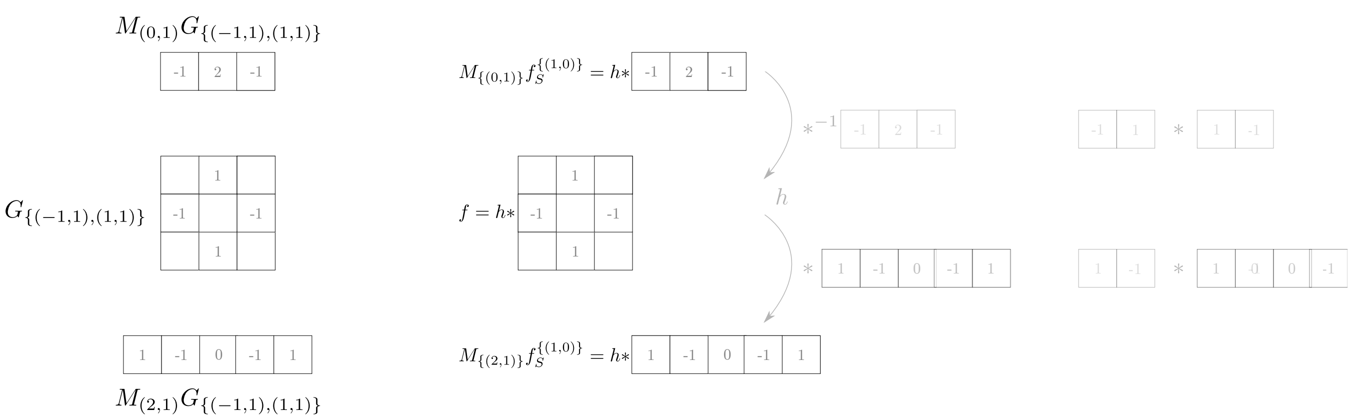

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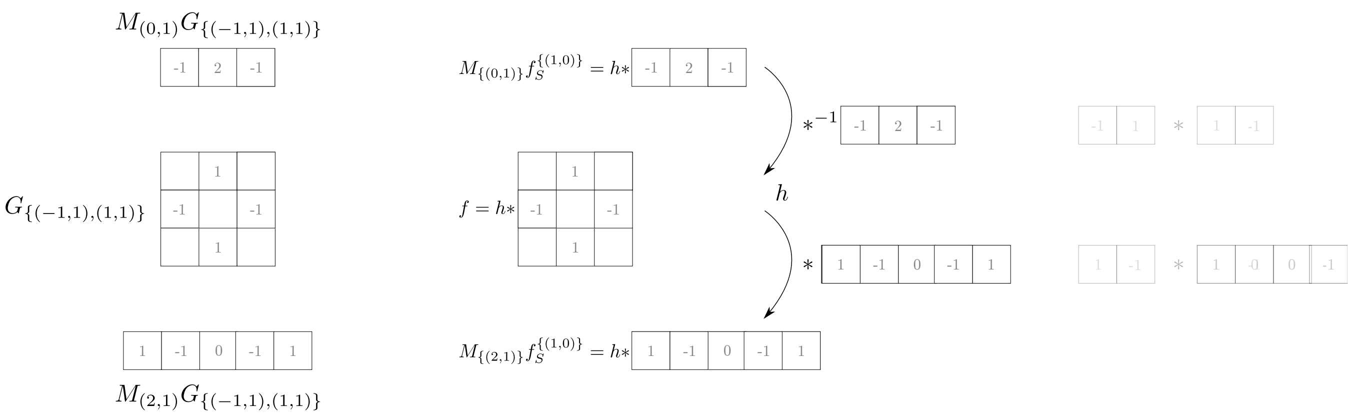

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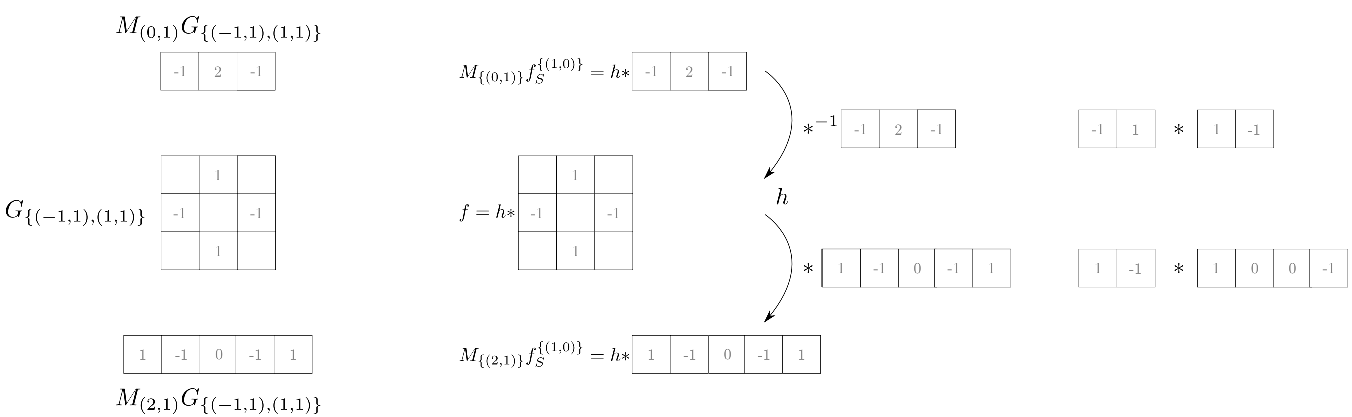

Application to Distributed Storage Systems

Distributed Storage Systems

Failures are inevitable

- Different origins: hardware, software, network, ...
- Can be simultaneous

The Mojette transform as an erasure code

- Provides the same fault-tolerance (e.g. 3 failures)
- Requires a significant lower amount of redundancy than plain replication
    - storage consumption is 1.5 for Mojette versus 3 for triplication

Embedded in RozoFS: a distributed file system

Distributed Storage Systems

Node repair (today by naïve solution)

- Failures are inevitable
- How to repair disks that suffer from permanent failures
- Re-projection along a previously existing projection direction

Dynamic fault-tolerance adaptation (not implemented)

- Adaptive need in fault-tolerance
- Re-projection along a new projection direction

Conclusion

Conclusion

1. Decomposition of the reconstruction process into partial reconstructions
2. Replace the reconstruction process by convolution operations

Open Question

- Is it possible to use a lower amount of projections?

Thanks !

RozoFS is opensource and is available here:

https://github.com/rozofs/rozofs

My contact is:

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