Research and development project

Mojette transform on regular non-cubic lattices

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Outline

Outline

Statement of the problem

- Study of the Mojette transform
- Mostly used on cubic lattices
- Study on non-cubic lattices
- Performance comparison between different lattices

Outline

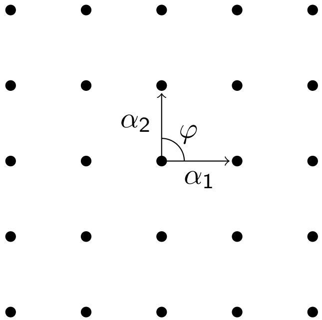
Objectives

- Study of the different lattices
- Selection of a non-cubic lattice
- Compare the performance of the Mojette transform between the cubic and non-cubic lattices
- Find some comparison criteria.

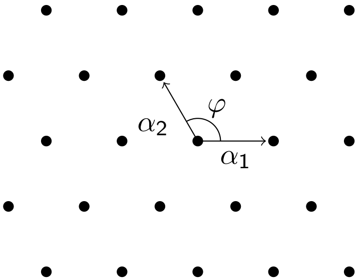
Outline

Lattices

What is a lattice ?



(a) The Square Lattice.
$|\alpha_1| = |\alpha_2|$, $\varphi = 90\,^\circ$.



(b) The Hexagonal Lattice.
$|\alpha_1| = |\alpha_2|$, $\varphi = 120\,^\circ$.

Figure: Representation of different lattices

Outline

Comparison criteria
Redundancy

The standard definition of redundancy is:

$$Red = \frac{nb_{bins}}{nb_{pixels}} - 1 \quad (1)$$

Comparison criteria

Bins number variance

We define the Bins number variance as:

$$Var(B) = \frac{1}{n} \sum_{i=1}^{n} (B_i - B_m)^2 \quad (2)$$

with

- n projections
- B_i the number of bins on projection i
- B_m average number of bins

Comparison criteria

Pixels per bin mean

We define the pixels per bin mean as:

$$Mean(pixels\ per\ bin) = \frac{1}{n}\sum_{i=1}^{n} b_i \tag{3}$$

with

- n the total number of bins
- b_i the value of the bin i

Comparison criteria
Pixels per bin variance

We define the pixels per bin variance as:

$$Variance(pixels\ per\ bin) = \frac{1}{n}\sum_{i=1}^{n}(b_i - Mean(pixels\ per\ bin))^2 \quad (4)$$

with

- n the total number of bins
- b_i the value of the bin i

Outline

Test case
Grids construction

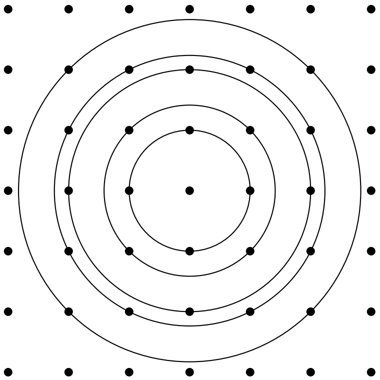


Figure: Square grid construction

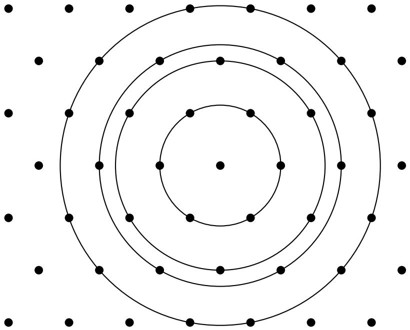


Figure: Hexagonal grid construction

Test case

Selection of projection

Theorem

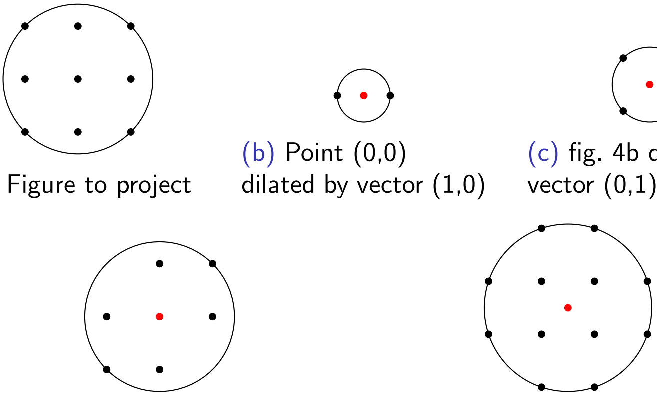
Complete reconstructability theorem: Statement 1: Both propositions are equivalent:

- $f(k, l)$ *defined on the convex* G *is reconstructible by* $\{proj_{p_i,q_i}, 1 \leq i \leq I\}$;
- R *constructed by* I *dilations ser* $\{O, (p_i, q_i), 1 \leq i \leq I\}$.

Statement 2: Both propositions are equivalent:

- G *is reconstructible by* $\{proj_{p_i,q_i}, 1 \leq i \leq I\}$;
- *the erosion of* G *by* R *is null.*

Test case
Selection of projection



(a) Figure to project

(b) Point (0,0) dilated by vector (1,0)

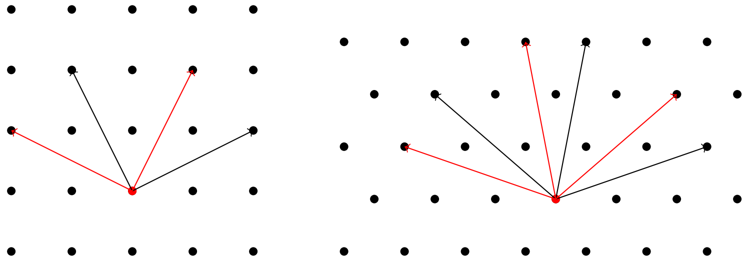
(c) fig. 4b dilated by vector (0,1)

(d) fig. 4c dilated by vector (1,1)

(e) fig. 4d dilated by vector (-1,1)

Figure: Example of dilatation in square lattice

Test case
Selection of projection



(a) rotation of a vector (2,1) in Square Lattice

(b) rotation of a vector (3,1) in Hexagonal Lattice

Figure: Rotation example of a direction

Outline

Results
Redundancy

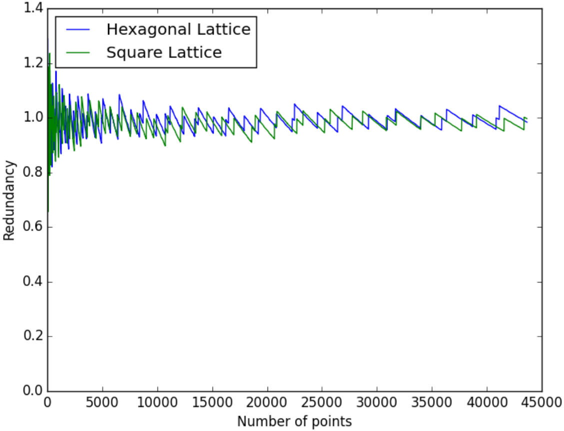


Figure: Redundancy

Outline

1. Introduction
   - Statement of the problem
   - Objectives
   - Lattices
2. Proposals
   - Comparison criteria
   - Test case
3. Results
   - Redundancy
   - Bins number variance
   - Pixels per bin mean
   - Pixels per bin variance
4. Conclusion

Results

Bins number variance

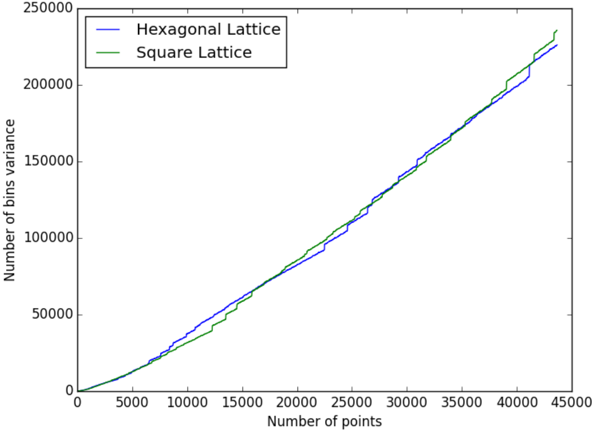


Figure: Bins number variance

Outline

Results

Pixels per bin mean

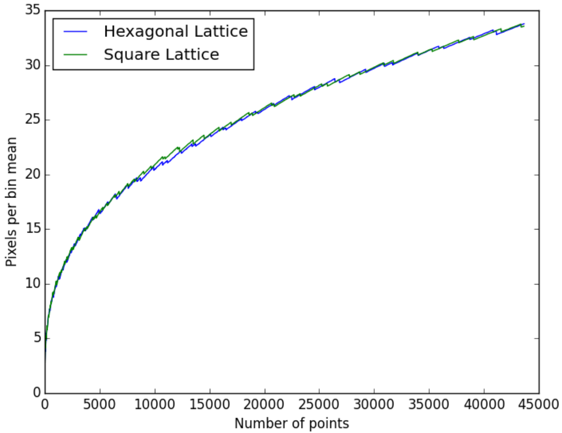


Figure: Pixels per bin mean

Outline

Results
Pixels per bin variance

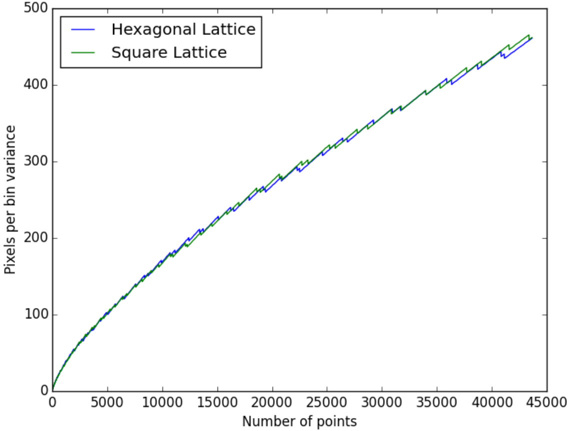


Figure: Pixels per bin variance

Conclusion

- These results are not conclusive
- study in 3D ?

Conclusion

Do you have any questions ?